

L6: Dynamics (I)

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Spring, 2021

Kinematics v.s. Dynamics

- *Kinematics* describes the motion of objects. We have been talking about rigid transformation and derivatives w.r.t. time.
- *Dynamics* describes the cause of motion. We will talk about mass, energy, momentum, and force.
- The basic law of dynamics, Newton's Law, describes the motion of a point mass:

$$\mathbf{f} = m\mathbf{a}$$

- But there are caveats that you may not be aware of.

Kinematics v.s. Dynamics

- We start from point mass dynamics and will move on to rigid body dynamics.
- We will provide certain proofs but not all (many are very tricky and lengthy).

A Tale of Three Frames

Concepts

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- **Observer's Frame:**

- When we record any motion, we choose the observer's frame \mathcal{F}_o , so that every point would have a coordinate and every vector will have a direction and length.
- For our symbols, this is on the superscript.
- If the frame is moving (e.g., taken to be the body frame), when recording motions, we first *clone a version* of this frame and *keep it static* for recording.

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- **Reference Frame:**

- When recording the movement of objects, we introduce a reference frame so that the notion of movement is *relative to* this frame.

Some Notes on Reference Frame

- Reference Frame:
 - When recording the movement of objects, we introduce a reference frame so that the notion of movement is *relative to* this frame.
- We have not discussed this frame much in developing robot **kinematics theories**.

In dynamics, the choice of reference frame is not arbitrary!

Recording a *Relative Velocity*

- We introduce $s(t)$ to denote a reference frame which may be moving.
- Then we denote the relative velocity as below:
- Relative velocity for a point mass
 - $\mathbf{v}_{s(t) \rightarrow b(t)}^o = \mathbf{v}_{o \rightarrow b(t)}^o - \mathbf{v}_{o \rightarrow s(t)}^o$
- Relative velocity for rigid body
 - $\boldsymbol{\xi}_{s(t) \rightarrow b(t)}^o = \boldsymbol{\xi}_{b(t)}^o - \boldsymbol{\xi}_{s(t)}^o$
- Consistency

$$\mathbf{v}_{s(t) \rightarrow b(t)}^o = \boldsymbol{\xi}_{s(t) \rightarrow b(t)}^o \mathbf{p}^o$$

where \mathbf{p}^o is a point observed in \mathcal{F}_o

Inertia Frame

- Inertia frame refers to the choice of the *reference frame*.
- Only in an inertia frame can Newton's law be written as $\mathbf{f} = m\mathbf{a}$.
- Definition of Inertia frame:
 - Where the law of inertia (Newton's First Law) is satisfied.
 - Any free motion has a constant magnitude and direction.
- A clear notion of Newton's Second Law:

$$\mathbf{f}^o = m\mathbf{a}_{s(t) \rightarrow b(t)}^o$$

where $s(t)$ is an inertia frame (o is static).

Fictitious Force

- What if the reference frame is not an inertia frame?
- Assume we have two moving frames, $\mathcal{F}_{s(t)}$ and $\mathcal{F}_{b(t)}$
 - e.g., the earth and an object sitting on the earth
- We are interested in how the force \mathbf{f}^o affects the relative acceleration $\mathbf{a}_{s(t) \rightarrow b(t)}^o$

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- Some intuition that $\mathbf{f}^o \neq m\mathbf{a}_{s(t) \rightarrow b(t)}^o$
 - Since $\mathcal{F}_{s(t)}$ is moving with an angular velocity, any object $b(t)$ moving along with it must also have an acceleration to gain the same angular velocity.
 - Computation shows that some additional force will be consumed to maintain the relative velocity of $b(t)$ against $s(t)$.

Fictitious Force

- Computing $\mathbf{f}^o = d(m\mathbf{v}_{s' \rightarrow b(t)}^o)/dt$ (note: s' is chosen to be an inertia frame), and we have

$$\mathbf{f}^o - m \frac{d\boldsymbol{\omega}^o}{dt} \times \mathbf{r}^o - 2m\boldsymbol{\omega}^o \times \mathbf{v}^o - m\boldsymbol{\omega}^o \times (\boldsymbol{\omega}^o \times \mathbf{r}^o) = m\mathbf{a}^o$$

where

- \mathbf{f}^o : the physical forces acting on the object
- $\boldsymbol{\omega}^o := \boldsymbol{\omega}_{s' \rightarrow s}^s$
- $\mathbf{v}^o := \mathbf{v}_{s(t) \rightarrow b(t)}^o$
- $\mathbf{r}^o := \mathbf{r}_{s(t) \rightarrow b(t)}^o$
- $\mathbf{a}^o := \mathbf{a}_{s(t) \rightarrow b(t)}^o$

Fictitious Force

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- Euler force: $-m \frac{d\boldsymbol{\omega}^o}{dt} \times \mathbf{r}^o$
- Centrifugal force: $-m\boldsymbol{\omega}^o \times (\boldsymbol{\omega}^o \times \mathbf{r}^o)$
- Coriolis force: $-2m\boldsymbol{\omega}^o \times \mathbf{v}^o$

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- Kinematics describes the motion of objects. We describe the motion using displacement and velocity and acceleration.
- Dynamics describes the forces that cause the motion. We will look at Newton's laws of motion and how they relate to kinematics.
- Dynamics is the study of the forces that cause the motion of objects.

$$F = ma$$

- The force is the cause of the acceleration.

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A Tale of Three Frames

Concepts

- Reference Frame: A set of axes and a clock used to describe the motion of objects. The origin of the frame is the point from which the motion is measured.
- Inertial Frame: A reference frame in which Newton's laws of motion hold. Inertial frames are those that are not accelerating.
- Non-Inertial Frame: A reference frame in which Newton's laws of motion do not hold. Non-inertial frames are those that are accelerating.

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Recording a Relative Velocity

- The velocity of an object in one reference frame is related to its velocity in another reference frame by the relative velocity of the frames.
- The relative velocity of two frames is the velocity of one frame as measured in the other frame.
- The relative velocity of two frames is denoted by \vec{v}_{rel} .
- The relative velocity of two frames is a vector.

$$\vec{v}_{rel} = \vec{v}_2 - \vec{v}_1$$

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